Exponenciální rovnice

Příklady řešené převodem na stejný základ

1) $5^3 \cdot 5^9 = \left(5^x\right)^3$

$5^{3+9} = 5^{3 \cdot x}$

$5^{12} = 5^{3x}$

$12 = 3x$

$x = 4$

$P = \{4\}$

2) $2^3 \cdot 4^{5x} = 8^{x+2}$ všechny základy mocnin nahradíme mocninou čísla 2

$2^3 \cdot \left(2^2\right)^{5x} = \left(2^3\right)^{x+2}$

$2^3 \cdot 2^{2 \cdot 5x} = 2^{3 \cdot (x+2)}$

$2^{3+10x} = 2^{3x+6}$

$3 + 10x = 3x + 6$

$7x = 3$

$x = \dfrac{3}{7}$

$P = \left\{\dfrac{3}{7}\right\}$

3) $3^x \cdot 9^{x-1} = 27$ všechny základy mocnin nahradíme mocninou čísla 3

$3^x \cdot \left(3^2\right)^{x-1} = 3^3$

$3^x \cdot 3^{2 \cdot (x-1)} = 3^3$

$3^{x+2x-2} = 3^3$

$x + 2x - 2 = 3$

$x = \dfrac{5}{3}$

$P = \left\{\dfrac{5}{3}\right\}$

4) $0,25^x \cdot 8 = 2^{5-x}$ všechny základy mocnin nahradíme mocninou čísla 2

$\left(2^{-2}\right)^x \cdot 2^3 = 2^{5-x}$

$2^{-2x} \cdot 2^3 = 2^{5-x}$

$2^{-2x+3} = 2^{5-x}$

$-2x + 3 = 5 - x$

$x = -2$

$P = \{-2\}$

5) $\left(5^x\right)^3 = 0,2^x \cdot 25$ všechny základy mocnin nahradíme mocninou čísla 5

$$5^{3x} = \left(5^{-1}\right)^x \cdot 5^2$$

$$5^{3x} = 5^{-x} \cdot 5^2$$

$$5^{3x} = 5^{-x+2}$$

$$3x = -x + 2$$

$$x = \frac{1}{2}$$

$$P = \left\{\frac{1}{2}\right\}$$

6) $\left(\frac{3}{4}\right)^{4x+2} = \left(\frac{4}{3}\right)^{2+3x}$ rovnice obsahuje převrácené zlomky: $\frac{4}{3}$ můžeme nahradit $\left(\frac{3}{4}\right)^{-1}$

$$\left(\frac{3}{4}\right)^{4x+2} = \left[\left(\frac{3}{4}\right)^{-1}\right]^{2+3x}$$

$$\left(\frac{3}{4}\right)^{4x+2} = \left(\frac{3}{4}\right)^{-2-3x}$$

$$4x + 2 = -2 - 3x$$

$$7x = -4$$

$$x = -\frac{4}{7}$$

$$P = \left\{-\frac{4}{7}\right\}$$

7) $2^{x-4} = \left(\sqrt{2}\right)^{2-3x}$

$$2^{x-4} = \left(2^{\frac{1}{2}}\right)^{2-3x}$$

$$2^{x-4} = 2^{\frac{1}{2} \cdot (2-3x)}$$

$$x - 4 = \frac{1}{2} \cdot \left(2 - 3x\right)$$

$$x - 4 = 1 - \frac{3x}{2} \quad / \cdot 2$$

$$2x - 8 = 2 - 3x$$

$$x = 2$$

$$P = \{2\}$$

8) $5^{2x+3} = 1$

$5^{2x+3} = 5^0$

$2x + 3 = 0$

$x = -\dfrac{3}{2}$

$P = \left\{-\dfrac{3}{2}\right\}$

9) $4^{-x} = 64$

$4^{-x} = 4^3$

$-x = 3$

$x = -3$

$P = \{-3\}$

10) $0,25^x = 16$

$\left(2^{-2}\right)^x = 2^4$

$2^{-2x} = 2^4$

$-2x = 4$

$x = -2$

$P = \{-2\}$

11) $100^{2x+1} = 0,01$

$\left(10^2\right)^{2x+1} = 10^{-2}$

$10^{2\cdot(2x+1)} = 10^{-2}$

$2\cdot(2x+1) = -2$

$4x + 2 = -2$

$x = -1$

$P = \{-1\}$

12) $\left(\dfrac{2}{3}\right)^{3x-7} = \left(\dfrac{3}{2}\right)^{7x-2}$

$\left(\dfrac{2}{3}\right)^{3x-7} = \left[\left(\dfrac{2}{3}\right)^{-1}\right]^{7x-2}$

$\left(\dfrac{2}{3}\right)^{3x-7} = \left(\dfrac{2}{3}\right)^{-7x+2}$

$3x - 7 = -7x + 2$

$10x = 9$

$x = \dfrac{9}{10}$

$P = \left\{\dfrac{9}{10}\right\}$

13) $2^{2x+1} = \left(\sqrt{2}\right)^{2x-1}$

$2^{2x+1} = \left(2^{\frac{1}{2}}\right)^{2x-1}$

$2^{2x+1} = 2^{\frac{1}{2}\cdot(2x-1)}$

$2x + 1 = \dfrac{1}{2}\cdot(2x-1)$

$2x + 1 = \dfrac{2x-1}{2} \quad / \cdot 2$

$4x + 2 = 2x - 1$

$x = -\dfrac{3}{2}$

$P = \left\{-\dfrac{3}{2}\right\}$

14) $4^3 \cdot 4^2 = \left(4^{x-1}\right)^4$

$4^{3+2} = 4^{4\cdot(x-1)}$

$5 = 4x - 4$

$x = \dfrac{9}{4}$

$P = \left\{\dfrac{9}{4}\right\}$

15) $25^{2x+1} \cdot 0,2^2 = 5^{-3+x}$

$\left(5^2\right)^{2x+1} \cdot \left(5^{-1}\right)^2 = 5^{-3+x}$

$5^{4x+2} \cdot 5^{-2} = 5^{-3+x}$

$5^{4x+2+(-2)} = 5^{-3+x}$

$4x = -3+x$

$x = -1$

$P = \{-1\}$

16) $\dfrac{2^{2x-1}}{2^{x+2}} = 4^{3x}$

$\dfrac{2^{2x-1}}{2^{x+2}} = 4^{3x} \quad / \cdot 2^{x+2}$

$2^{2x-1} = \left(2^2\right)^{3x} \cdot 2^{x+2}$

$2^{2x-1} = 2^{6x} \cdot 2^{x+2}$

$2^{2x-1} = 2^{6x+x+2}$

$2x - 1 = 7x + 2$

$x = -\dfrac{3}{5}$

$P = \left\{-\dfrac{3}{5}\right\}$

17) $4 \cdot 0,25^{3x+1} = 8^{2-x}$

$2^2 \cdot \left(2^{-2}\right)^{3x+1} = \left(2^3\right)^{2-x}$

$2^2 \cdot 2^{-6x-2} = 2^{6-3x}$

$2^{2+(-6x)+(-2)} = 2^{6-3x}$

$2 - 6x - 2 = 6 - 3x$

$x = -2$

$P = \{-2\}$

18) $\dfrac{1}{3^{5-2x}} = 81$

$\dfrac{1}{3^{5-2x}} = 81 \quad / \cdot 3^{5-2x}$

$1 = 3^4 \cdot 3^{5-2x}$

$3^0 = 3^{4+5-2x}$

$0 = 4 + 5 - 2x$

$x = \dfrac{9}{2}$

$P = \left\{ \dfrac{9}{2} \right\}$

19) $\sqrt{2^{2x}} = 8$

$\left(2^{2x} \right)^{\frac{1}{2}} = 2^3$

$2^x = 2^3$

$x = 3$

$P = \{3\}$