Logaritmické rovnice

Logaritmické rovnice typu „logaritmus = logaritmus“

1) $\log_2(5x+2) = \log_2(4x+3)$

2) $\log_3(2x+4) - \log_3(4x-1) = 0$

3) $\log_6 3 + \log_6(2x-5) = \log_6(x-1)$

4) $\log_3(2x-1) + \log_3(3x+2) = \log_3(7x-2)$

5) $\log_6(7x+2) - \log_6 5 = \log_6(x+5)$

6) $\log(17x-6) - \log(x+3) = \log 8$

7) $\log_5(2x+4) - \log_5(x-2) = \log_5(x+2)$

8) $2\log(x-2) = \log(14-x)$

9) $2 \cdot \log(x+3) - \log(2-2x) = 0$

10) $\dfrac{\log(2x+10)}{2} = \log(x+1)$

11) $\log(54-x^3) = 3 \cdot \log x$

Řešení

1)

$\log_2(5x+2) = \log_2(4x+3)$

$5x+2 = 4x+3$

$x = 1$ vyhovuje

$P = \{1\}$

2)

$\log_3(2x+4) - \log_3(4x-1) = 0$

$\log_3(2x+4) = \log_3(4x-1)$

$2x+4 = 4x-1$

$-2x = -5$

$x = \frac{5}{2}$ vyhovuje

$P = \left\{\frac{5}{2}\right\}$

3)

$\log_6 3 + \log_6(2x-5) = \log_6(x-1)$

$\log_6\left[3(2x-5)\right] = \log_6(x-1)$

$6x-15 = x-1$

$5x = 14$

$x = 2,8$ vyhovuje

$P = \{2,8\}$

4)

$\log_3(2x-1) + \log_3(3x+2) = \log_3(7x-2)$

$\log_3\left[(2x-1)(3x+2)\right] = \log_3(7x-2)$

$(2x-1)(3x+2) = 7x-2$

$6x^2 + 4x - 3x - 2 = 7x - 2$

$6x^2 - 6x = 0$

$x \cdot (6x-6) = 0$

$x_1 = 0$

$x_2 = 1$

Vyhovuje pouze $x = 1$

$P = \{1\}$

5)

$\log_6(7x+2) - \log_6 5 = \log_6(x+5)$

$\log_6 \frac{7x+2}{5} = \log_6(x+5)$

$\frac{7x+2}{5} = x+5 \quad / \cdot 5$

$7x+2 = 5x+25$

$2x = 23$

$x = 11,5$ vyhovuje

$P = \{11,5\}$

6)

$\log(17x-6) - \log(x+3) = \log 8$

$\log \frac{17x-6}{x+3} = \log 8$

$\frac{17x-6}{x+3} = 8 \quad / \cdot x+3$

$17x-6 = 8x+24$

$9x = 30 \quad / : 9$

$x = \frac{10}{3}$ vyhovuje

$P = \left\{\frac{10}{3}\right\}$

7)

$\log_5(2x+4) - \log_5(x-2) = \log_5(x+2)$

$\log_5 \frac{2x+4}{x-2} = \log_5(x+2)$

$\frac{2x+4}{x-2} = x+2 \quad / \cdot (x-2)$

$2x+4 = (x+2) \cdot (x-2)$

$2x+4 = x^2 - 4$

$x^2 - 2x - 8 = 0$

$D = (-2)^2 - 4 \cdot 1 \cdot (-8) = 36$

$x_1 = \frac{2+\sqrt{36}}{2} = 4 \qquad x_2 = \frac{2-\sqrt{36}}{2} = -2$

Vyhovuje pouze $x = 4$

$P = \{4\}$

8)

$2\log(x-2)=\log(14-x)$

$\log(x-2)^2=\log(14-x)$

$(x-2)^2=14-x$

$x^2-4x+4=14-x$

$x^2-3x-10=0$

$D=b^2-4ac=(-3)^2-4\cdot1\cdot(-10)=49$

$x_1=\frac{-b+\sqrt{D}}{2a}=\frac{3+7}{2}=5$

$x_2=\frac{-b-\sqrt{D}}{2a}=\frac{3-7}{2}=-2$

Vyhovuje pouze $x=5$

$P=\{5\}$

9)

$2\cdot\log(x+3)-\log(2-2x)=0$

$2\cdot\log(x+3)=\log(2-2x)$

$\log(x+3)^2=\log(2-2x)$

$(x+3)^2=2-2x$

$x^2+6x+9=2-2x$

$x^2+8x+7=0$

$D=8^2-4\cdot1\cdot7=36$

$x_{12}=\frac{-8\pm\sqrt{36}}{2\cdot1}$

$x_1=-1$

$x_2=-7$

Vyhovuje pouze – 1

$P=\{-1\}$

10)

$$\frac{\log(2x+10)}{2} = \log(x+1) \quad / \cdot 2$$

$$\log(2x+10) = 2 \cdot \log(x+1)$$

$$\log(2x+10) = \log(x+1)^2$$

$$2x+10 = (x+1)^2$$

$$2x+10 = x^2 + 2x + 1$$

$$x^2 = 9$$

$$x_1 = 3$$

$$x_2 = -3$$

Vyhovuje pouze $x = 3$

$$P = \{3\}$$

11)

$$\log(54 - x^3) = 3 \cdot \log x$$

$$\log(54 - x^3) = \log x^3$$

$$54 - x^3 = x^3$$

$$54 = 2x^3$$

$$x^3 = 27$$

$x = 3$ vyhovuje

$$P = \{3\}$$